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J. Guckenheimer, D. Armbruster (postdoc) and S. Campbell (grad student) and I have continued our work on the global dynamics and bifurcations of O(2) symmetric ODEs. Such systems are obtained as finite dimensional projections or reductions of spatially translation- and reflection-invariant PDE"S, for example. In 1987/88, partially supported by this grant, we provided a complete analysis of heteroclinic cycles and modulated travelling waves in two mode (k:2K) interacting systems [1,2]. In particular, we pointed out that heteroclinic cycles are structurally stable features in such systems [cf.3].

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AFOSR 84-0051 Final Report

P.Holmes. Research Activities January 1, 1988 - September 31, 1988

1. Dynamics of O(2) Symmetric Systems

J. Guckenheimer, D. Armbruster (postdoc) and S. Campbell (grad student) and I have continued our work on the global dynamics and bifurcations of O(2) symmetric ODEs. Such systems are obtained as finite dimensional projections or reductions of spatially translation- and reflection-invariant PDE's, for example. In 1987/88, partially supported by this grant, we provided a complete analysis of heteroclinic cycles and modulated travelling waves in two mode (k:2k) interacting systems [1,2]. In particular, we pointed out that heteroclinic cycles are structurally stable features in such systems [cf.3].

The original suggestion that interesting global dynamical behavior arises in O(2) symmetric problems came from work on boundary layers [4], where the heteroclinic cycles correspond to bursting behavior. In that system five or more "modes" are involved and we are therefore now studying a three mode (k:2k:4k) interaction, which appears to capture the essence of a crucial interaction mechanism in the five mode boundary layer model. The system of ODEs in six dimensional, having 3 complex (Fourier) modes, and so presents a formidable analytical challenge. Armbruster and Campbell have performed extensive numerical simulations and uncovered pervasive families of modulated travelling waves. We are currently studying various types of limiting systems, for small parameter regimes near the multiple bifurcation point at which all three modes are neutrally stable, as well as attempting to approximate Poincare return maps near the heteroclinic cycles. We hope to continue this work under a new grant.

2. Spatially Complex Equilibria of Buckled Rods

The work done by A. Mielke and me in 1987 has been published [5] and these studies are now complete. Using "Melnikov" perturbation methods and exploiting the noncanonical Hamiltonian structure of the equilibrium equations for a hyperelastic rod, we were able to provide explicit computations demonstrating the existence of spatially "chaotic" equilibrium shapes. Such solutions are the spatial analogue of the chaotic dynamics proven to occur in the dynamical problems of a rotating rigid body subject to gravity (cf.[6], Kirchoffs' kinetic analogy).

References

1 D. Armbruster, J. Guckenheimer and P. Holmes, Physica D 29: 257-282, 1988. Heteroclinic cycles and modulated travelling waves in systems with O(2) symmetry.

2________________________, SIAM J.. on Appl. Matter (in press) 1988. Kuramoto-Sivashinsky dynamics on the Center Unstable Manifold.

3 J. Guckenheimer and P. Holmes, Math Proc. Camb. Phil. Soc. 103: 189-192, 1988. Structurally Stable Heteroclinic Cycles.

4 N. Aubry, P. Holmes, J. Lumley and E. Stone. J. Fluid Mech 192: 115-173, 1988. The dynamics of coherent structures in the wall region of a turbulent boundary layer.

5 A. Mielke and P. Holmes, Arch. Rat. Mech. Anal. 101:319-348, 1988. Spatially complex equilibria of buckled rods.

6. A.E.H. Love. A Treatise on the Mathematical Theory of Elasticity, Cambridge, 1927.

Invited Lectures and Seminars on work supported by this grant:
Heteroclinic Cycles and Modulated Travelling Waves in O(2) Symmetric Systems.

University of St.Andrews, Scotland	5/11/88
DAMPT, Cambridge University, England	5/19/88
Mathematics Institute, Warwick University, England	5/23/88
CNLS, Los Alamos National Lab, NM	8/23/88

R.Rand. Research Activities January 1, 1988 - September 31, 1988

1. Averaging using Jacobian Elliptic functions

We have, together with graduate student Vincent Coppola, investigated the dynamics of the system

(1) $$x'' + \alpha(\tau)\, x + \beta(\tau)\, x^3 = \epsilon\, g(x,x',t)$$

in which $\tau = \epsilon t$ is slow time. The idea here is that when $\epsilon = 0$ the system reduces to

(2) $$x'' + \alpha(0)\, x + \beta(0)\, x^3 = 0$$

which has a solution in terms of Jacobian elliptic functions, e.g.

(3) $$x = A\, \mathrm{cn}(\varphi,k)$$

We have applied the method of averaging to first and second orders in ϵ to the system (1). This involves perturbing off of Jacobian elliptic functions, rather than off of trignometric functions as is usually done. The resulting equations involve integrals of elliptic functions which are evaluated using a program written in the computer algebra system MACSYMA. The results have been applied to the problem of finding limit cycles in the autonomous version of eq.(1) [1], as well as to the chaotic behavior of the eq.:

(4) $$x'' + x \cos\tau + x^3 = 0$$

The origin in this equation is saddle-like for half of its period, and is center-like for the other half. By analyzing the slow flow associated with eq.(4),

(5) $$A' = -\frac{\epsilon}{A} \sin\tau \left[1 - \frac{E(k)}{K(k)}\right]$$

$$\varphi' = \frac{(A^2 - \cos\tau)^{1/2}}{4K(k)}$$

we have been able to prove the existence of chaotic behavior in the eq.(4).

2. Computer Algebra Treatment of the Elliptic Restricted 3 Body Porblem

We have, together with graduate student Bhaskar Viswanadham, investigated the stability of the L_4 equilibrium solution in the elliptic restricted problem of three bodies. This problem is governed by the following d.e.'s.

(2.1) $$x'' - 2y' - g\, h_2\, x = 0$$

(2.2) $$y'' + 2x' - g\, h_1\, y = 0$$

where $g = 1/(1 + e\cos f)$ and

$$h_{1,2} = \frac{3}{2}\left[1 \pm [1 - 3\mu(1-\mu)]^{1/2}\right]$$

in which primes represent differentiation with respect to the true anomaly f (the independent variable). Eqs.(2) involve two parameters μ (mass ratio of the smaller primary to the sum of the primaries) and e (eccentricity of the orbit of the primaries). For given μ,e, the equilibrium L_4 is stable if all solutions to (1) are bounded, and unstable if an unbounded solution exists.

Although numerous papers have been written since the mid-1960's on this problem, we have gone far further in the perturbation analysis than previous investigators by the use computer algebra and an efficient perturbation algorithm. We obtained 45 terms in the perturbation series for the transition curves separating regions of stability in the μ-e plane from regions of instability, with the result that studies of the convergence of the series have been made possible. It was found that one of the series exhibits unusual behavior in that it can be shown to converge, but not to the correct (i.e., numerically-obtained) solution [2].

<u>References and Recent Publications:</u>

1. Symbolic Computation and Perturbation Methods using Elliptic Functions
V.T.Coppola and R.H.Rand
abstract in Sixth Army Conference on Applied Mathematics and Computing, May 31-June 3, 1988, University of Colorado at Boulder

2. Computer Algebra and the Elliptic Restricted Problem of Three Bodies
Richard H. Rand and Bhaskar Viswanadham
to appear in the Proceedings of the 1989 Canadian Congress of Applied Mechanics (CANCAM)

3. Lie Transforms Applied to a Nonlinear Parametric Excitation Problem
J.L.Len, R.H.Rand
in Transactions of Fifth Army Conference on Applied Mathematics and Computing, ARO Report 88-1, 1-27 (1988)

4. Systems of Coupled Oscillators as Models of Central Pattern Generators
R.H.Rand, A.H.Cohen and P.J.Holmes
Chapter 9 in Neural Control of Rhythmic Movements in Vertebrates, Ed. A.H.Cohen, 333-368
John Wiley & Sons (1988)

5. Computer Algebra Implementation of Lie Transforms for Hamiltonian Systems: Application to the Nonlinear Stability of L_4
V.T.Coppola and R.H.Rand
abstract in Sixth Army Conference on Applied Mathematics and Computing, May 31-June 3, 1988, University of Colorado at Boulder

6. Subharmonic Entrainment of a Forced Relaxation Oscillator
D. Storti, R.H. Rand
Int.J.Nonlinear Mechanics <u>23</u>:231-239, 1988

7. Degenerate Homoclinic Cycles in Perturbations of Quadratic Hamiltonian Systems
J. Guckenheimer, R. Rand, D. Schlomiuk
submitted for publication in Nonlinearity

8. Perturbation Methods and Computer Algebra
R.H. Rand
to appear as a Chapter in <u>The Use of Symbolic Methods to Solve Algebraic and Geometric Problems Arising in Engineering</u>, ed. R. Grossman (SIAM)

9. The Use of Symbolic Computation in Perturbation Analysis
R.H. Rand
to appear in Proceedings of 1988 ASME Winter Meeting

F.Moon. Research Activities January 1, 1988 - September 31, 1988

1. Chaotic Vibrations of a Three-Well Potential Oscillator

Together with postdoctoral research associate G. Li, we have studied the forced vibrations of a nonlinear elastic beam having three stable and two unstable equilibrium positions. The responses of the system exhibit chaotic behavior in certain regions in the driving frequency and amplitude plane. With the use of the Melnikov technique, the homoclinic and heteroclinic bifurcations of the system were calculated. Numerical simulations of the stable and unstable manifolds, Poincare maps, Lyapunov exponents, and fractal basin boundaries were performed. The results indicate that both homoclinic and heteroclinic bifurcations are necessary conditions for chaos, and that the final steady state of motion for many initial conditions becomes unpredictable if at least one homoclinic or heteroclinic bifurcation occurs. An experimental criterion and a heuristic formula for chaotic motion were obtained and compared with the bifurcation criteria.

2. Chaotic Nonplanar Motions of a Forced Linearly Elastic Rod.

Together with graduate student J.Cusumano we have combined a geometrically exact rod theory with new analytical and experimental techniques from dynamical systems theory to explain complex dynamical phenomena observed in a thin prismatic steel rod with clamped-free boundary conditions. The support of the rod is harmonically displaced with the axis of the displacement aligned with the lateral axis of symmetry of the rod so that one would expect motions to remain planar. However, experiments show that planar motions become unstable when the forcing frequency is close to a resonant frequency of the system. The resulting motions are nonplanar and chaotic.

A previously undiscovered family of asymmetric bending-torsion nonlinear modes have been found experimentally. Other phenomena discovered in the system include dynamic two-well behavior and energy cascading from high frequency input to low frequency response. The fractal dimension of the attracting sets have been estimated directly from experimentally obtained scalar time series.

The resulting dimension estimates are below 5, implying that it may be possible to model the dynamics of the rod with as few as two degrees of freedom. A geometrically exact, linear elastic rod theory is developed, and used to derive a model system of two nonlinearly coupled pde's, which is then further approximated by two ode's by the assumed-modes method. A family of nonlinear modes analogous to those found experimentally in the rod were discovered numerically. The behavior of the model has been favorably compared to the experimental data.

Recent Publications

F.C. Moon, G.-X. Li: "Chaotic Dynamics in Magnetic and Magneto-Mechanical Systems", The IUTAM Symposium on "The Electromagnetic-Mechanical Interactions in Deformable Solids and Structures", Tokyo, Japan, October 12-17, 1986.

F.C. Moon, J. Cusumano, P.J. Holmes: "Evidence for Homoclinic Orbits as a Precursor to Chaos in a Magneto Pensulum", Physica **24D**, pp. 383-390, 1987.

F.C. Moon, G.-X. Li: "Chaotic Dynamics in Magnetic and Magneto-Mechanical Systems", Electromagneto-mechanical Interactions in Deformable Solids and Structures, Y. Yamamoto, K. Miya, Editors, North-Holland, 1987, pp. 41-52.

F.C. Moon: "Chaotic Dynamics in Solid Mechanics", to appear in New Directions in Nonlinear Mechanics, F. Salam, Editor.

G.-X. Li, F.C. Moon: "Multiple Homoclinic Bifurcation Criteria for Chaos for a Two-Degree of Freedom Nonlinear Oscillator", submitted to Physical Review Letters.

F.C. Moon, J.D. Wang, R. Raj: "High T_c Superconducting Magnetic Bearings for Levitation of High Speed Rotors", submitted to Science.

F.C. Moon, M.M. Yanoviak, R. Ware: "Hysteretic Levitation Forces in Supeconducting Ceramics," Appl. Phys. Lett. **52**, 1534-1536 (1988).

F.C. Moon: "Mag-Lev Transport for the 21st Century", remarks by F.C. Moon before the Senate Hearing of Senator D.P. Moynihan, Thursday, January 21, 1988.

F.C. Moon: "Experiments in Chaotic Dynamics". a summary of lectures given at the International Center for Mechanical Sciences (CISM) at Udine, Italy, July 1986.

F.C. Moon, G. Raggio: "Chaotic Scattering of Waves from Nonlinear Scatterers", submitted to Acous. Soc. America.

F.C. Moon, K.-C. Weng, P.-Z. Chang: "Dynamic Magnetic Forces in Supeconducting Ceramics", submitted to Appl. Phys. Lett.

F.C.Moon, John R. Hull, Greg F. Berry: "Supeconductivity and Mechanical Engineering."

F.C. Moon, Guangxuan Li: "Experimental Study of Chaotic Vibrations in a Pin-Jointed Space Truss Structure."

Guangxuan Li, F.C. Moon: "Chaotic Vibrations of a Nonlinear Elastic System with Homoclinic and Heteroclinic Orbits."

M.P. Paidoussis, F.C. Moon: "Nonlinear and Chaotic Fluidelastic Vibrations of a Flexible Pipe Conveying Fluid", submitted to 1988 Congress of IUTAM.

C.-K. Lee, F.C. Moon: "Laminated Piezopolyme Plates for Torsion and Bending Sensors and Actuators", submitted to J. Acoust. Soc. Am.

M.P. Paidoussis, Guangxuan Li, F.C. Moon: "Chaotic Oscillations of the Autonomous System of a Constrained Pipe Conveying Fluid",

F.C. Moon: "Chaotic Vibrations of a Magnet near a Superconductor", submitted to Phys. Lett. A.

C.-K. Lee, F.C. Moon: "Modal Sensors/Actuators", submitted to ASME J. Appl. Mech.

Guangxuan Li, F.C. Moon: "Vibrations of a Three-Well Potential Oscillator with Homoclinic and Heteroclinic Orbits", submitted to J. Sound and Vibration.

F.C. Moon, John R. Hull, Greg F. Berry: "Superconductivity: As Temperaturs Rise, so do Demands on MEs", Mechanical Engineering **110**, June 1988.

F.C. Moon: "Cornell Superconducting Bearing Research, June 1988.

F.C. Moon: Chaotic Vibrations, John Wiley and Sons. 1987